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432. Proposed by C. N. SCHMALL, New York City.

There are n straight lines in a plane, no two of which are parallel and no three of which are concurrent. Their points of intersection being joined show that the number of new lines drawn is $\frac{1}{8}n(n-1)(n-2)(n-3)$.

SOLUTION BY LAENAS G. WELD, Pullman, Illinois.

Each of the n given lines intersects each of the other $(n-1)$ lines, determining (since each point is thus twice determined) points to the number of $\frac{n(n-1)}{2}$.

Let P_{ik} be the point determined by the lines l_i and l_k . From P_{ik} there may be drawn $R \left(= \frac{n(n-1)}{2} - 1 \right)$ lines to the other points. But the line l_i contains $(n-2)$ of these other points and the line l_k the same number. Hence the given lines l_i and l_k account for $2(n-2)$ of the above R lines and the number of new lines is

$$R - 2(n-2) = \left(\frac{n(n-1)}{2} - 1 \right) - 2(n-2).$$

Since there are $\frac{n(n-1)}{2}$ points such as P_{ik} the total number of new lines (since each is thus counted twice) is

$$\frac{1}{2} \cdot \frac{n(n-1)}{2} \left\{ \left(\frac{n(n-1)}{2} - 1 \right) - 2(n-2) \right\},$$

or

$$\frac{1}{8}n(n-1)(n-2)(n-3).$$

Also solved by J. W. CLAWSON, HERBERT N. CARLETON, H. C. FREEMSTER, WALTER C. EELLS, PAUL CAPRON, FRANK R. MORRIS, ELBERT H. CLARKE, N. P. PANDYA, and HORACE OLSON.

GEOMETRY.

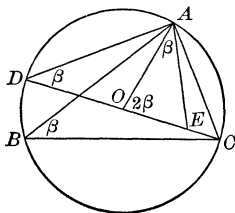
461. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Prove by means of any inscribed triangle the following trigonometrical relations:

$$\sin 2\beta = 2 \sin \beta \cos \beta; \quad \cos 2\beta = \cos^2 \beta - \sin^2 \beta; \quad \sin 3\beta = 3 \sin \beta - 4 \sin^3 \beta; \\ \cos 3\beta = 4 \cos^3 \beta - 3 \cos \beta.$$

SOLUTION BY J. W. CLAWSON, Collegeville, Pa.

Let ABC be the triangle. Join A and C to O , the center of the circumcircle. Extend CO to meet the circle at D . Draw AD . Draw AE making angle OAE equal to angle ABC . Denote angle ABC by β . Call the radius R .



$$(1) \quad \sin 2\beta = \sin AOC = \sin ACD \cdot \frac{AC}{R} = \frac{AC \cdot AD}{2R^2} = 2 \frac{AC}{2R} \cdot \frac{AD}{2R} = 2 \sin \beta \cos \beta.$$

$$(2) \quad \cos 2\beta = \cos AOC = \frac{R^2 + R^2 - \overline{AC}^2}{2R \cdot R} = \frac{4R^2 - \overline{AC}^2}{4R^2} = \frac{\overline{AD}^2 - \overline{AC}^2}{4R^2} \\ = \left(\frac{AD}{2R} \right)^2 - \left(\frac{AC}{2R} \right)^2 = \cos^2 \beta - \sin^2 \beta.$$

$$(3) \quad \sin 3\beta \sin AEC = \sin AEO = \sin \beta \cdot \frac{R}{OE}.$$

Now

$$\frac{OE}{R} = \frac{AE}{AD} = \frac{AE}{2R \cos \beta}; \quad i. e., AE = 2\overline{OE} \cos \beta.$$

Again

$$OE^2 = \overline{AE}^2 + R^2 - 2\overline{AE} \cdot R \cos \beta = 4\overline{OE}^2 \cos^2 \beta + R^2 - 4\overline{OE} \cdot R \cdot \cos^2 \beta;$$

$$\therefore OE = \frac{R}{4 \cos^2 \beta - 1};$$

$$\therefore \sin 3\beta = \sin \beta (4 \cos^2 \beta - 1) = 3 \sin \beta - 4 \sin^3 \beta.$$

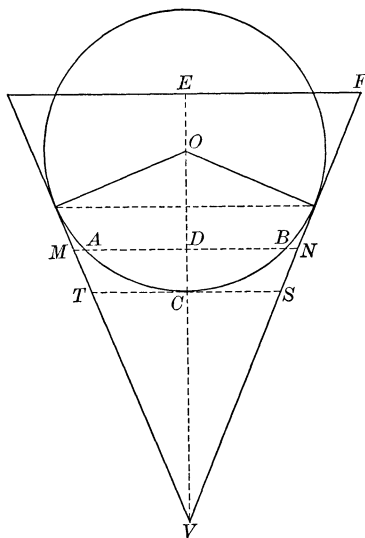
$$(4) \quad \cos 3\beta = \frac{R^2 - \overline{AE}^2 - \overline{OE}^2}{2\overline{OE} \cdot AE} = \frac{(4 \cos^2 \beta - 1)^2 - 1 - 4 \cos^2 \beta}{4 \cos \beta} = 4 \cos^3 \beta - 3 \cos \beta.$$

462. Proposed by DANIEL KRETH, Wellman, Iowa.

A conical glass, the diameter of the base of which is 5 inches and altitude 6 inches, is one-fifth full of water. If a sphere 4 inches in diameter is dropped into it, how much of the vertical axis of the glass is immersed?

SOLUTION BY J. A. CAPRON, Notre Dame University.

Let MN be the level of the water after the sphere is dropped; $R = 5/2$ inches = radius of base of given cone; $r = 2$ inches = radius of sphere; r_1 = radius of base of cone VMN ; x = height of spherical segment ACB ; α = semivertical angle of cone; and $CV = a$.



Then volume of cone $VMN = V_c = \frac{1}{3}\pi r_1^2(a+x)$; volume of segment $ACB = V_s = \pi x^2(r-x/3)$; and volume of given cone $= V = \frac{1}{3}\pi R^2 h$, where $h = 6$ inches.